Modeling and Efficiency Analysis of Multi-Phase Resonant Switched Capacitive Converters

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***Abstract*—This paper presents an analytical method to evaluate pertinent data of the resonant capacitive switching converter especially the voltage gain and power efficiency. Instead of long transient simulation time, the proposed model uses frequency decomposition to speed-up computation. This method is valid for N-phase operation and extends the recently published studies on this promising topology outside zero-current/voltage switching conditions. Thanks to this tractable expression, we also reveal the intrinsic efficiencies over the voltage gain of 2- and 3-phase structures working at the resonant frequency in step-down operation. These results help to gain better understanding of multi-phase operation and encourage additional studies to use the full capability offered by the resonant switched capacitor converter especially for power on-chip integration.**

***Keywords*— DC-DC converter; switched-mode power supplies; resonant.**

I. INTRODUCTION

The Switched Capacitor Converter (SCC) approach has been well-studied in the literature over the past two decades [1]. The main advantage is to allow a highly integrated power supply compatible with IC technology for granular power management. However, the SCC topology suffers from inherent low efficiency when the output voltage is not a fraction of the input voltage [2]. The Resonant Switched Capacitor Converter (RSCC) has been described in recent years by [3]–[11]. The main benefits of this structure are to limit the flying-capacitor charging loss of the SCC counterpart and to operate under soft switching conditions (i.e. zero-current/voltage switching ZCS/ZVS) thus reducing the switching loss. Promised power-density is achieved by [12] to design an on-chip point-of-load voltage regulator. In the literature, only 2- or 3-phase RSCCs have been described and modeled under ZCS/ZVS operation. This paper extends the model to N- phase RSCCs without any switching condition by an alternative approach. Then, we explore its potential benefit outside ZCS/ZVS operation especially for the wide-range lossless regulation, one of the key issues in SCC and RSCC structures. First, we detail the frequency-domain analysis of the N-phase RSCC providing pertinent data such as the conversion ratio and power efficiency which do not need a large processing capacity. Based on the proposed model, we explore 2- and 3-phase operation. This paper shows that the 3-phase is able to reach better efficiency compared to the SCC counterpart in step-down configuration under the same ideal conditions.

II. N-PHASE RESONANT SWITCHED CAPACITOR CONVERTER MODEL

The N-phase RSCC formed by a j-switch network SW_j and RLC tank is shown in Figure 1. The inductor is much smaller than in a conventional inductive-based converter because the main energy is stored in the capacitor [13]. The N-phase, denoted ϕ_i, is completed in one resonant period $T = \sum_{i=1}^{N} T_{\emptyset i} = 2\pi\sqrt{LC}$ called a cycle. A switching sequence (one cycle) is illustrated in Fig. 1 [14]. There are four possible switch states called S_k shown in Table 1 for each phase based on the switch configuration of a 2:1 SCC (K_{in} and K_{out} coefficients are used later).

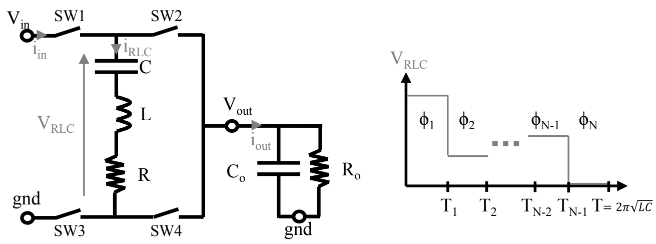

Fig. 1. Resonant switched capacitor converter.

To study the achievable efficiency of an N-phase RSCC in this letter, the assumptions for the proposed model are i) negligible on-state resistance of the switches, ii) no parasitic component in the RLC tank e.g. no bottom plate, iii) the period of the N-phase called a cycle is equal to the resonant frequency, iv) the quality factor of the RLC tank is sufficiently high, v) the load is modeled as a voltage source (the bypass capacitor C_o is sufficiently large) and vi) steady-state operation. The effect of the control scheme, the interleave configuration and the switching loss are however beyond the scope of this letter.

TABLE 1. THE FOUR SWITCH STATES OF AN RSCC.

State	V_{RLC}	SW_1	SW_2	SW_3	SW_4	$K_{in} = \frac{i_{in}}{i_{rlc}}$	$K_{out} = \frac{i_{out}}{i_{rlc}}$
S_1	V_{in}-V_{out}	ON	OFF	OFF	ON	1	1
S_2	V_{out}	OFF	ON	ON	OFF	0	-1
S_3	0	OFF	ON	OFF	ON	0	0
S_4	V_{in}	ON	OFF	ON	OFF	1	0

Considering the assumptions described above and by using Fourier transform, the frequency decomposition of the voltage across the RLC tank V_{RLC} can be expressed as:

$$i_{RLC}(t) = \frac{V_{RLC}(t)}{Z} = \frac{a_0}{Z(0)} + \sum_{n=1}^{+\infty} \frac{a_n \cos nwt + b_n \sin nwt}{Z(nw)}$$

$$\underset{\text{assumption}}{\overset{\text{under iv)}}{\approx}} \frac{a_1 \cos wt + b_1 \sin wt}{R} \tag{1}$$

where the Fourier coefficient a_1 and b_1 are equal to:

$$a_1 = \frac{2}{T}\int_{t=0}^{T} V_{RLC}(t) \cos wt\, dt = \frac{2}{T}\sum_{i=0}^{N-1}\int_{t=T_i}^{T_{i+1}} V_{RLC,\phi i} \cos wt\, dt \tag{2}$$

$$b_1 = \frac{2}{T}\int_{t=0}^{T} V_{RLC}(t) \sin wt\, dt = \frac{2}{T}\sum_{i=0}^{N-1}\int_{t=T_i}^{T_{i+1}} V_{RLC,\phi i} \sin wt\, dt \tag{3}$$

where N is the number of phases, T_i is the i^{th} switching instant between the i^{th} and $i+1^{th}$ states, T is the resonant period within which the cycle is completed ($T_{\phi N}=T$), and $V_{RLC,\phi i}$ is the voltage across the RLC tank in the i^{th} switching phase (defined in Table 1).

The averages of the input and output currents are then given by:

$$\langle i_{in}\rangle = \frac{1}{T}\sum_{i=0}^{N-1}\int_{T_i}^{T_{i+1}} K_{out,\phi i}\, i_{RLC}(t)dt \tag{4.a}$$

$$\langle i_{out}\rangle = \frac{1}{T}\sum_{i=0}^{N-1}\int_{T_i}^{T_{i+1}} K_{in,\phi i}\, i_{RLC}(t)dt \tag{4.b}$$

where $K_{in,\phi i}$ and $K_{out,\phi i}$ are the coefficients in the ith phase defined in Table 1.

By combining (1-4), the voltage gain and power efficiency are expressed using the design variables T_i, R and R_o so that:

$$\alpha = \frac{V_{out}}{V_{in}} = \frac{R_o\langle i_{out}\rangle}{V_{in}} \tag{5.a}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{R_o\langle i_{out}\rangle^2}{V_{in}\langle i_{in}\rangle} \tag{5.b}$$

The achievable efficiency given by (5.b) does not depend on the switching frequency. This result is also confirmed in the following section in (8) and (9) for 2- and 3-phase operation. The switching frequency implying switching loss (neglected here) only sets the LC tank value, then the RSCC power density. An increase in frequency, reduces the LC value and size but also increases the switching loss thus decreasing the efficiency. Therefore, we are in line with the scope of this letter which focuses on the theoretical achievable efficiency of N-phase RSCC under ideal conditions i.e. ideal switches.

The method could be extended to N-harmonic decomposition if the quality factor is high enough to neglect the harmonics as shown in [5]. For the clarity of this paper, the next results are given using only the fundamental decomposition but the complete expression was calculated using third harmonic extension to check their effect.

III. Derivation of Method to 2-Phase Operation

To illustrate the simplicity of the model derivation, we propose a calculation for a 2-phase 2:1 RSCC. The first and second phases are S_1 and S_2, respectively, as in 2:1 SCC. The Fourier coefficients of the i_{RLC} are calculated from (2) and (3) so that:

$$a_1 = \frac{2}{T}\left(\int_{T_o=0}^{T_1}(V_{in}-V_{out})\cos wt\,dt + \int_{T_1}^{T_2=T} V_{out}\cos wt\,dt\right) = \frac{V_{in}}{\pi}(1-2\alpha)s_1 \tag{6}$$

$$b_1 = \frac{2}{T}\left(\int_{T_o=0}^{T_1}(V_{in}-V_{out})\sin wt\,dt + \int_{T_1}^{T_2=T} V_{out}\sin wt\,dt\right) = \frac{V_{in}}{\pi}((2\alpha-1)c_1 + 1 - 2\alpha)$$

where $c_i = \cos(wT_i)\,, s_i = \sin(wT_i), \alpha = \frac{V_{out}}{V_{in}}$

Then, the input and output currents are found from (4) and using the i_{RLC} expression, we obtain:

$$\langle i_{out}\rangle = \frac{1}{T}\left(\int_{T_o=0}^{T_1} i_{RLC}dt + \int_{T_1}^{T_2=T_{sw}} -i_{RLC}dt\right) = \frac{1}{\pi R}(b_1 + a_1 s_1 - b_1 c_1)$$
$$\langle i_{in}\rangle = \frac{1}{T}\int_{T_o=0}^{T_1} i_{RLC}dt = \frac{1}{2\pi R}(b_1 + a_1 s_1 - b_1 c_1) \tag{7}$$

Using (5) and the previous results (6-7), the voltage gain and power efficiency are thus:

$$\alpha = \frac{V_{out}}{V_{in}} = \frac{s_1^2 + c_1^2 - 2c_1 + 1}{2s_1^2 + 2c_1^2 - 4c_1 + 2 + \pi^2\frac{R}{R_o}} < 0.5 \tag{8.a}$$

$$\eta = \frac{P_{out}}{P_{in}} = 2\alpha < 1 \tag{8.b}$$

The numerical resolution of equation (8) has been verified against full circuit simulation under the same conditions and shows less than 1% variation. The power efficiency expressed in (8) is linearly proportional to the voltage gain as previous work has already described [4]. Figure 2(a) shows the best efficiency for a 2-phase RSCC and the associated optimal switching instant T_1. For example, if $R \ll R_o$ and $T_1=T/2$, the voltage gain is equal to 0.5 and the process is theoretically lossless ($\eta=1$). These results prove that the maximal efficiency of a 2-phase RSCC is achieved when $\alpha=0.5$ as in a classical SCC. A lower voltage gain implies a lower efficiency equal to the 2:1 SCC counterpart. Expression (8) also highlights the effect of the parasitic resistance R compared to the load resistor R_o which limits the achievable efficiency. In Fig. 2(a) the ratio R_o/R is fixed at 10 and 100 to illustrate the tank resistor effect. The highest efficiency in an RSCC is limited only by the resistor ratio instead of the inherent charging-loss (proportional to C×F factor) in an SCC. The efficiency is insensitive to the LC value and resonant frequency as long as the conduction and switching losses are not taken into account in our analysis (see the aforementioned assumptions). Fig. 2(b) shows the time-domain waveforms of the RLC tank and output for a 0.3 voltage gain. The proposed model highlights an unexpected switching sequence. For example, the converter works in hard-switching conditions to reduce the loss, and the output current i_{out} is sometimes negative to average the output charge transfer in the resonant period T.

The $\{S_1,S_2\}$ RSCC cycle, also called 2:1 operation, cannot achieve flatness efficiency versus voltage gain as is mathematically proven in (8) even if all components are ideal (no on-state resistance, no resistance in the LC tank). We have also analyzed other possible cycles limited to 2 phases. In this case, the cycle could be equal to $\{S_k,S_{k'}\}$ where $\{k,k'\}\varepsilon[1,2,3,4]$ means 42 different cycles. We have used a symbolic computation software (Matlab associated with a symbolic toolbox) to solve equations (1) to (5) for each cycle. Unfortunately, there is no configuration where the efficiency is not linearly proportional to the voltage gain in step-down configuration.

IV. ANALYSIS OF 3-PHASE OPERATION

To alleviate the efficiency dependence on the voltage gain, some papers [4], [5] propose to add an extra phase. However, adding more phases implies more switching losses but it almost disengages the efficiency from the voltage gain. Here, we propose to study this 3-phase operation by keeping the cycle time equal to the resonant period (where the presented method is valid). Under this constraint, we have analyzed the $\{S_1,S_2,S_3\}$ sequence for a 3-phase RSCC. In this particular sequence, the voltage gain and power efficiency of the cycle are given following the same method as in 2-phase i.e. by:

$$\alpha = \frac{2s_1^2 + 2c_1^2 - s_1 s_2 - c_1 c_2 - 3c_1 + c_2 + 1}{4s_1^2 + s_2^2 + 4c_1^2 + 2c_2 - 4c_1 c_2 - 4c_1 + 2c_2 + 1 + 2\pi^2 \frac{R}{R_o}} \quad (9)$$

$$\eta = \frac{2\pi^2 R}{R_o} \frac{\alpha^2}{(1-2\alpha)(s_1^2 + c_1^2) + \alpha(s_1 s_2 + c_1 c_2) - (3\alpha - 2)c_1 - \alpha c_2 - \alpha + 1}$$

Even if these analytical equations do not directly show the efficiency dependence on the voltage gain as in (8), their numerical resolution helps to rapidly find the optimal efficiency by exploring the design free variables T_1 and T_2 from 0 to unity where $T_2>T_1$ ($T_3=T=1$ as a cycle is completed within the normalized resonant period). Figure 3(a) gives the best efficiency for a 3-phase RSCC and the associated optimal switching instants. The association of ideal 1:1 and 2:1 SCCs where their efficiencies are equal to α and 2α, respectively, is also drawn as a dashed line. The result is computed in a few seconds on a laptop, many decades faster than full transient simulations. Full circuit simulations have also been performed to compare the results with the analytical expressions. The mismatch is always less than 1%. Figure 3.b plots the time-domain waveforms for a 0.3 voltage gain at the optimal switching instants $\{T_1,T_2\}$ given by Fig. 3(a). As for 2-phase operation, the optimal sequence on the one resonant period leads to unexpected behavior in the time-domain.

As can be seen from Fig.3, the 3-phase RSCC with $\{S_1,S_2,S_3\}$ cycle maintains a better efficiency compared to the 2-phase SCC in step-down configuration ($\alpha<1$). Even if all components are ideal, the simulation shows that it is not possible to maintain a constant efficiency over the voltage gain by keeping the cycle frequency at that of resonance. We have also taken into account the second and third harmonic components in the previous calculation but we have observed a slight difference when $R<<R_o$.

The RSCC in hard switching condition but the overall RSCC efficiency is maximized compared to other possible switching instants under the assumptions described in Section I. One limitation of the proposed operation is to create some EMI issues due to a current spike problem [15] and additional switching loss at high switching operation.

V. CONCLUSION

In this paper, we give a generic method to analyze any cycle sequence of a multi-phase RSCC. We also address the inherent power efficiency limits of 2- and 3-phase RSCCs operating at the resonant frequency outside soft-switching conditions. The model is based on frequency-domain decomposition especially allowing fast exploration of its inherent efficiency capability. We have demonstrated the model by 2- and 3-phase converters to find the tractable analytical expressions of their voltage gain and power efficiency. Excellent agreement was found between the proposed model and the classical transient simulation. The fast resolution of the expression allows us to determine the best switching instants to reach the highest efficiency for any voltage gain in a step-down configuration. Thus, we confirm a more constant efficiency RSCC profile over a wide range of

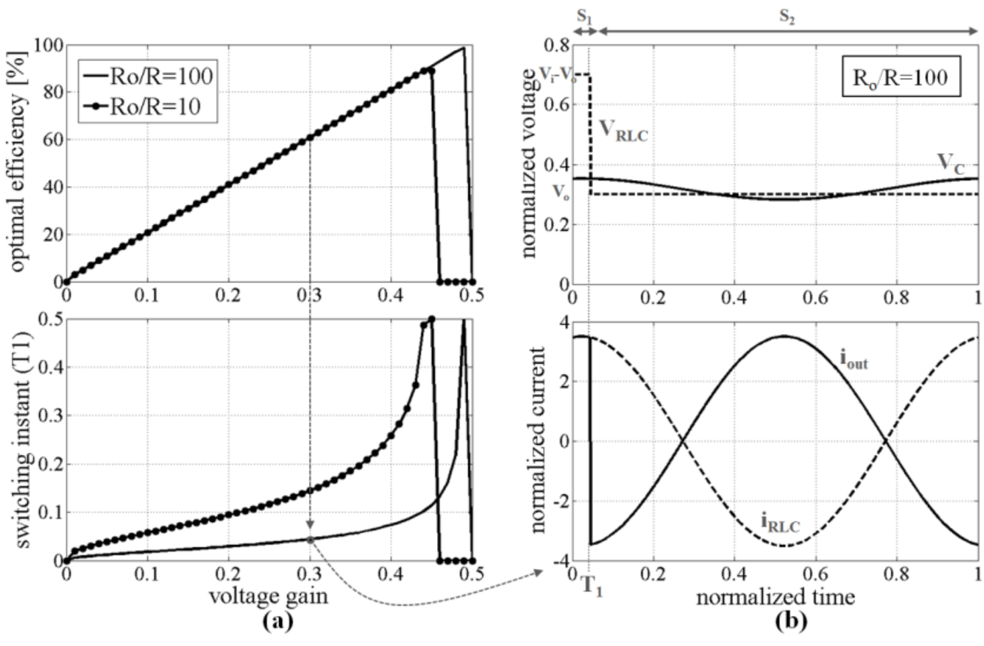

Fig. 2. (a) Efficiency v. voltage gain of a 2-phase RSCC $\{S_1,S_2\}$ at resonant frequency, (b) time-domain waveforms at a 0.3 voltage gain.

voltage gain compared to a conventional SCC when a third phase is added. Although the discussion has been applied to 2- and 3-phase resonant converter efficiency, the method is valid regardless of tank impedance, other switch structure or for evaluating other pertinent data in the circuit. The model could also be extended to outside resonant frequencies by taking into account the other harmonics but the analytical expressions are less tractable. However, the model has fundamental limitations based on the assumptions given in section I. These limitations and extension to 4 phases will be the direction of further research.

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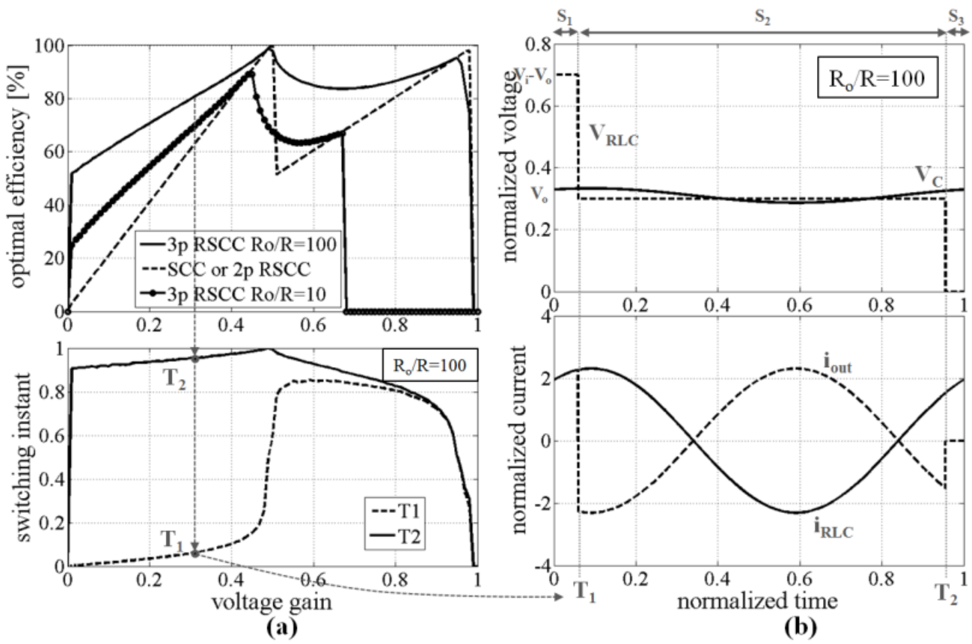

Fig. 3. (a) Efficiency v. voltage gain of a 3-phase RSCC $\{S_1,S_2,S_3\}$ at resonant frequency compared to a 2-phase RSCC and SCC, (b) time-domain waveform at a 0.3 voltage gain.